Future Directions For Studying The Learning And Teaching Of Algebra: Lessons From The Past

Bill Atweh, Center Mathematics & Science Education,
Tom J Cooper, Center Mathematics & Science Education
Gillian Boulton-Lewis, School Learning & Development

Queensland University of Technology

This paper argues for a future direction for studying the learning and teaching of algebra based on collaborative action research between teachers and university researchers. It considers the results of two different types of research undertaken from social and cognitive perspective of the algebra classroom. This research has shown that, regardless of learning, algebra tends to act as a stratification agent for the social reproduction of economic differences, necessitating alternative approaches to existing research.

The 1980s have witnessed a rising interest in research on school algebra culminating in two major publications by the National Council for Teachers of Mathematics (Coxford & Schulte, 1988; Wagner & Kieran, 1989). This research identified major problems in the learning and teaching of algebra (e.g., Booth, 1988; Kieran, 1992). Research in this area has continued into the 1990s (e.g., McGregor & Stacey, 1995) as evidenced by an increase in national and international conferences, competitive grants and publications. There have been new initiatives in instruction (e.g., Quinlan, Low, Sawyer, & White, 1993), but there is little evidence that problems in teaching and learning of algebra have been solved on a large scale. The future of research on, and arguably teaching and learning of, algebra is still open.

From a simple curriculum perspective, algebra could be considered as facing a future similar to computation. Technology, predominantly in the form of calculators, has acted on computation in three ways. First, it has offered itself as a replacement for the algorithms that used to make up the primary computation syllabus. Second, it has provided rapid accurate computation power that has opened up the applications of computation. Third, it has provided powerful teaching aids to assist in the teaching of concepts, principles and thinking strategies that underlie computation. As a consequence to this, the curriculum emphasis in computation is turning from algorithmic procedures to number sense and problem solving. Technology is also providing options for the algorithmic procedures that make up the secondary algebra syllabus (e.g., solving equations, simplifying expressions, drawing graphs, differentiating and integrating functions). It is also opening up new vistas for algebra applications (e.g., the algebra needs of apprentice electronic engineers). It is also providing excellent teaching aids to assist students to discover the concepts and principles behind, e.g., variable, equation and function. These topics appear to offer a rich basis for teaching understanding and the structure of the relationships and operations on these concepts provides a rich field for principles. Thinking of algebra as the language of generalisation seems to complement this approach to algebra and leads to a curriculum emphasis, similar to computation, on algebra sense and applications and modelling. However, the analysis that arrives at this conclusion is artificial in its restriction to curriculum issues and cause-effect argument. It fails to take into account the social consequences of algebra instruction and the proactive nature of learners' cognition.

In this paper, we shall consider the future of research in algebra that is likely to inform effective changes in practice. We argue that in order to discern what type of research is needed in the future, we need to look at what kind of research we have constructed in the past, what questions we have asked and what methodologies we have utilised. Ideas in this paper are grounded in reflection and self-critique about our own

involvement in two research projects on algebra as well as other types of research. Our aim is not to present a comprehensive research agenda, but rather to provoke thinking about possible new directions for research that may prove to be beneficial for solving some of the problems in school algebra.

Project One: Social context of algebra teaching

In the first study, we observed four Year 9 classrooms (second year of the secondary school in Queensland) for the duration of a single unit of work which dealt with techniques for solving linear equations (Atweh & Cooper, 1995, Cooper, Atweh, Baturo, & Smith 1993). The classes came from four different private schools with differing socio-economic backgrounds and gender of students. All teachers followed the same chapter from the same textbook.

Our observations and interviews have demonstrated that each of the teachers had constructed certain images of their students' needs and abilities in algebra. These perceptions were in the same direction predicted by previous research findings that formal school subjects are more appropriate for students from high socio-economic backgrounds and for many career aspirations of males. Further, student-teacher interactions in the four classrooms differed in accordance with these constructed perceptions. For example, the emphasis on rigor and formal language was more evident in the high socio-economic boys' school, the emphasis on multiple ways of solving equations and understanding was evidenced in the high socio-economic girls' school, the emphasis on algebra as only accessible to more able students was evidenced in the low socio-economic boys' school, while the low socio-economic girls' school participated in what we have described as a *ritual* of covering the curriculum topic that was not seen as relevant to the current or the future lives of the girls.

In all our publications stemming from this project, we emphasised the significant differences that we observed in the different classes. We also hinted that there were striking similarities in teaching approaches. All classes were quite traditional in their approach to teaching and learning, and dominated by the teacher who solved sample problems and allowed time for student practice of lengthy exercises. A slight exception to this was observed at the high socio-economic girls' schools that utilised group learning to a limited extent. It appeared to us that the differences that we have observed were not related to learning theories adhered to by the different teachers, but to the perceptions that the teachers have developed on the type of students that they have in that school.

This project had its limitations. It did not attempt to investigate why and how these perceptions arose , even less to attempt to change them. Lastly we were never sure that the views that we saw in the four classroom corresponded to those of the teachers involved. We do not believe that the teachers or the students directly benefited from our research.

However, the results of the social context research were in one way unequivocal. Regardless of learning, the algebra acted as a direct agent for social reproduction. The practice of teaching and learning algebra in the low socio-economic girls' class reinforced the position, that they had already accepted, that abstract mathematics such as algebra, and the job opportunities that go with it, was not for them. Like the 'lads' in Willis's study (1977), they built their own classroom subculture as a response to their decision not to take algebra learning seriously. The low socioeconomic boys' learning was much harsher. Their teacher directly used the algebra to separate the class into those who could do it and those that could not. The school's emphasis on abstract learning meant that the only option for most boys was failure. There appeared to be no subculture to maintain self esteem in the face of this failure. For the high socioeconomic boys' class, the algebra reinforced their belief that they were the elite - they were being groomed for professional jobs and that this type of formal and rigorous algebra was for them. The high socioeconomic girls' class was not so straightforward. The teachers' position meant that although the girls learnt that, for them, success in algebra was expected, they also learnt that they could achieve this success in a cooperative manner and that algebra was not the only way they could achieve success in life.

Mathematics as the agent for social stratification is not ã new position. Stake and Easley's (1978) research indicated this, as did the research of Anyon (1981). However, it is not a position that curriculum developers seem to have taken into account and it is central to the question of the future of algebra. Algebra may have utility, i.e., be necessary for a variety of vocations, and to improve life chances; it may have elegance as the language of generality; it may be central to the culture of our society and an important component of enculturation. However, in reality, it appears that its major role is to identify the minority who will be offered a chance at structural privilege.

Project Two: Use of manipulative materials and cognitive load

In this study, we observed a Year 8 class and individually interviewed students (7 girls and 14 boys) before and after instruction in linear equations using manipulative materials to develop solutions (Boulton-Lewis, Cooper, Atweh, Pillay, Wilss, & Mutch, 1995) The class was of mixed abilities from a middle socio-economic background school with a reputation for high academic standards and innovative mathematics teaching. The class has completed previous units which used patterns to introduced the concept of variable as generalisation and cups and counters to introduce variable as an unknown number. The unit under consideration used cups and counters to solve linear equations (using the methods of Thompson, 1988, including different coloured cups and counters to represent negative variables and numbers).

Similar to the social context research, the results in this study were unequivocal at one level - the students did not appear able to use the knowledge taught to them about the concrete representations. Only one of the twenty-one students correctly used cups and counters to represent the equation. At the post-interview, no students voluntarily used materials. When directly asked to use them, only four of the twenty-one students could use the materials to generate an answer. When given the choice students preferred a mental approach (based on inverse) which met their needs simply and effectively. As a result most uses of concrete representations were illustrative because mental strategies overrode any knowledge of the generative use of materials. The students did not appear to connect the teacher's material representation to their own mental representations.

The reason for this appears to lie within the processing loads associated with the particular concrete representations used in this class (Boulton-Lewis, 1993) and transferring understandings from arithmetic to algebra (Halford & Boulton-Lewis, 1992). The difficulty for students is that they have to integrate knowledge of laws and relations of arithmetic, knowledge of the mathematical meaning of equals and equation, knowledge of variable, and knowledge of methods to concretely represent variables, numbers and operations by cups and counters to solve the linear equation by materials.

Perhaps the reflections on this study that are relevant to the arguments developed here relate to the context of the research project. The school was carefully selected. We wanted to study the effect of using a concrete approach to algebra in a naturalistic environment. Very few schools in Brisbane use teaching approaches in algebra based on concrete materials. We did not wish to perform a teaching experiment and hence we did not wish to interfere with the normal context of the classroom. Even though the teacher was very eager to get some input for the researchers about his teaching, we were only ready to discuss our observations with him after the study had been completed. At that stage, we faced a great difficulty discussing our observations with him without making him feel that he was being assessed on the success of his teaching. Hence, like project one reported above, this study had its limitations as well. Our own knowledge of some of the problems in teaching algebra may have increased. We are not as confident as to why the teacher decided to implement this particular approach to algebra nor how to empower him to become more reflective practitioner and critical researcher to improve his practice - which he desired. Once again, we doubt that our research had directly benefited the teaching at that school.

Some Characteristics of Past Research on Algebra

The effect of cognitive/psychological constructs to study algebra is well documented. Kieran and Wagner (1989) identified the major factors effecting the teaching of the subject during this century from the task analysis work of Thorndike, to the meaningful learning movement of Brownell, through the developmental theory of Piaget and the more recent constructs of information processing theorists. The authors summarise their short history by observing: "a growing research interest in the factors and processes involved in the learning of school algebra merging from two directions, from cognitive psychologists and from mathematics educators" (p.5).

Kieran and Wagner were reporting on the deliberations of the 1987 conference convened at the University of Georgia as part of the Research Agenda Project which included "mathematicians, mathematics educators, psychologists, technologists, researchers, practitioners, and curriculum developers" (p. 5-6). Presumably "practitioners" meant, or at least included, classroom teachers. No paper presented to the conference was co-authored by school teachers. Likewise, of notable absence are sociological or anthropological perspectives. To be fair, the conference did conclude by raising the need to combine theory and practice and the role of multiple perspectives to study algebra.

> Some of the ideas that emerged from this exchange [during the concluding session of the conference] concerned the two -way relationship between theory and practice and the need to build theories based on the experience of practitioners. It was also suggested that new theory needs to be constructed in order to attempt to tie together the results of research from different traditions and to be able to predict how algebra learning takes place. (p.9)

It seems to us that past researchers in algebra have made certain assumptions about the nature and role of research knowledge and its relation to practice. We argue, that these assumptions are not conducive to the improvement of practice by research findings.

First, using the natural science models of knowledge generation and application, mathematics education seems to have developed the belief that knowledge about algebra learning and teaching should/could best be developed and decontextualised by controlled observations and experimentations by qualified and trained researchers. Further, knowledge developed from different perspectives converges to more comprehensive truths about algebra learning. When this knowledge is verified and validated, it can then be transferred to practitioners by "professional development" and used to improve their practice. Entrenched in the majority of research traditions in mathematics education is the separation of practice of knowledge generation and knowledge utilisation. This separation is graphically represented by Silver in his chapter on the role of research on practice in the 1990 NCTM Yearbook. After challenging the viewing of research as a search for the magic cure for problems of teaching and learning he stated that another metaphor "for thinking about the influence of educational research might be 'osmosis' the general permeation of the field of educational practice by ideas and constructs from the field of educational research and vice versa" (p.1). Silver outlined the main ways in which practitioners can utilise knowledge and methods in educational research. However, in the last paragraph of the chapter Silver did discuss the need for "researcher and practitioners to become collaborators in investigating issues of practical importance for the improvement of teaching and learning of mathematics"(p.9).

Second, behind much of the research in mathematics education is the belief that knowledge in ultimately incremental and convergent. Various studies in algebra education have isolated a number of factors and/or used predetermined constructs to study the teaching and learning of algebra. Viewing the classroom as a holistic complex micro-culture is a new awareness in mathematics education (Bauersfeld, 1992). In Western scientific thinking the study of complexity is believed to be possible by breaking it to small parts, and hence, an expert is seen as one "who knows more and more about less and less". The Gestalt argument that the whole is more than the sum of the parts is often

not reflected in educational research. A practitioner, on the other hand, has to function in a complexity. Laboratory knowledge generated by "expert researchers" often has to be balanced by real contexts, resource and structural limitations, teachers' and students' beliefs about the task at hand, and parents' and employers' expectations. Very few research studies have looked at the teaching and learning of algebra from such a complex perspective.

Third, the criteria of objectivity in research findings is being challenged. Much past research in algebra teaching and learning has adopted the stance that the problems that students face in the learning of algebra are due to either mathematical or psychological factors and hence can be overcome by pedagogical changes. Much research seem to concentrate on what the individual student can or cannot do. Students are often said to have not developed understanding even though they were able to demonstrate skill in manipulating algebraic equations to solve familiar problems. Much research reported in the Research agenda includes problems that students have in algebra, mathematical analysis of what algebra is and why is it difficult. Of notable absence from the conference report are discussions of motivation (including values) and algebra as a social construct. Knowledge generated often is a function of the questions raised (Apple, 1981). For example if research does not ask what makes some students successful in pursuing algebra in spite of negative attitudes and/or beliefs towards it, then our knowledge is limited. Evolving paradigms in educational research are raising new possibilities for new questions and methodologies, and are generating new understanding of teaching and learning mathematics. In particular, research from critical theory perspective has raised serious questions about the role of school algebra and everyday life mathematics (Harris, Lave)

In short, past research in algebra can be characterised by being based on the possibility of separation of theory and practice, on the principle that knowledge is additive and by limitation in questions that it raises.

Possible Directions for Research in the Future

The reflection above on our projects and research of others has raised some concern about the gap that exists between knowledge generation and knowledge use in traditional algebra research. The difference between the schools used in the social context project and the school observed for the cognitive study was in their teaching methods. The two studies had demonstrated that this was not sufficient to solve problems of teaching and learning algebra. Both studies have concentrated to look at factors that prevent effective learning. However, because of their design, both studies had limitations in suggesting and empowering change in practice.

We do not claim here to propose a complete research agenda for the future of school algebra. However, we believe that a line of research that may prove useful for improving the teaching and learning of algebra is critical Participatory Action Research (Kemmis & McTaggart, 1988).

> As I see it, action research aims to help people to investigate reality in order to change it (Fals Borda, 1979), and, at the same time, it also aims to help people to change reality in order to investigate it. In particular, action research attempts to help people investigate and change their social and educational realities by changing some of the practices which constitute their lived realities . . . .
> Through action research, people can come to understand their social and educational practices more richly, by locating their practices, as concretely and precisely as possible, in the particular material, social and historical circumstances within which their practices were produced, developed, and evolved - so that their real practices become accessible to reflection, discussion and reconstruction as products of past circumstances which are capable of being modified in and for present and future circumstances. While recognising that every practice is transient and evanescent, and that it can only be conceptualised in the inevitably abstract (though comfortingly

> imprecise) terms that language provides, action researchers aim to understand their own particular practices as they emerge in their own particular circumstances, without reducing them to the ghostly status of the general, the abstract, or the ideal - or, perhaps one should say, the unreal. (Kemmis, 1995, pp.)

We believe that through such a collaborative participatory and emancipatory action research approach the following may be possible.

- A question that intrigues us is: with the changes in curriculum emphasis around the world, the advancement of knowledge about algebra, and the technological demands, why is school algebra changing so slowly? An action research approach to study the problems of teaching and learning algebra aims at empowering teachers to critically reflect on their practice and to understand it thus becoming able to change it. Such research rejects the separation between practical and theoretical knowledge.
- The understanding the world in order to change it and changing the world in order to understand it principle advocated here results in the involvement of a more holistic knowledge that is more sensitive to the context in which that knowledge evolves hence more effective in changing the practices involved. It involves questions of cognition, but understands cognition in its wider social, not only personal and psychological, sense.
- Algebra is often used a critical filter for access to the privileges of society particularly to opportunities that do not use algebra or use a different type of algebra to that taught in schools. The approach to research advocated here by necessity has to raise questions of social justice aspects of algebra. Attempts to make the curriculum the same, and to have the same meaning, to all students is not only impossible from a cognitive learning perspective but also undesirable from a social justice agenda that is based on respect for difference rather than the distribution model of equal opportunity.

Such collaborative research takes into account the beliefs and conceptions of students, teachers and researchers (Tait & Boulton-Lewis, 1993). The first study highlighted the different social purposes for teaching algebra, while the second study identified some cognitive processing loads arising from certain teaching strategies. Hence, students resistance to learning algebra could be either to teaching methods or to perceived value of the task. Therefore the proposed approach here teachers, students and researchers may need to articulate their intentions and beliefs in order to understand the practice that they are involved in. In conjunction with the researchers, planned actions may be designed, implemented and evaluated. Perhaps though this process the cycle of disempowerment may be broken.

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